flux, W/m^2; α, heat-transfer coefficient, $W/m^2 \cdot °C$; w, velocity of heat-transfer-agent flow, m/sec; d,r, equivalent diameter and radius of channel, m; l, length of heat-transfer section, m; R, determining dimension (thickness) of all, m; Nu, Re, Pr, Fo, Nusselt, Reynolds, Prandtl, and Fourier numbers; L, initial temperature distribution function; A, E, Γ, dimensionless functions; C, B, constants. Indices: f, w, fluid (liquid) and wall; 0, initial value; 1, 2, heat-transfer surfaces; k, i, calculational and current time intervals.

LITERATURE CITED

1. L. D. Kalinnikov, "Experimental investigation of heat transfer in horizontal rectangular channel with small reduced thermal-section length," in: Automatic Control Systems in the Petroleum and Gas Industries (Proceedings of the All-Union Scientific-Research and Design Institute of Complex Automation of the Oil and Gas Industry) [in Russian], No. 5, Nedra, Moscow (1973).
2. N. V. Shumakov and L. D. Kalinnikov, "Correspondence of heat-transfer coefficients in steady and nonsteady processes," Inzh.-Fiz. Zh., 25, No. 5 (1973).
3. G. N. Kruzhilin, "Thermal boundary layers," Zh. Tekh. Fiz., 6, No. 3 (1936).
4. B. S. Petukhov, Heat Transfer and Drag in Laminar Fluid Flows in Tubes [in Russian], Énergiya, Moscow (1967).
5. L. D. Kalinnikov, "Solution of inverse heat-conduction problem with piecewise-linear approximation of boundary conditions," Teplofiz. Vys. Temp., 9, No. 5 (1971).

ALGORITHM FOR CALCULATING TEMPERATURE FIELDS IN THIN-WALLED STRUCTURAL ELEMENTS

V. S. Khokhulin

UDC 533.24.02

An algorithm for calculating temperature fields in thin-walled structural elements is considered which is based on the concept of local one-dimensional schemes in conjunction with graphical solution of problems in heat conduction.

In investigating the thermal regime of various structures, one often encounters the problem of calculating temperature distribution in support elements having complex configuration as a rule. To calculate the temperature distribution in these elements, the method of finite elements, which is based on a study of the thermal balance in the elementary volumes into which an element is divided, is the method mainly used. Calculation of the thermal balances in the selected volumes is a laborious and tedious problem for which the solution is of a specific nature in each case.

An attempt was made [1] at universalization of the methods for computing multidimensional temperature fields in structures. The method discussed in that paper finds application in the investigation of temperature fields of various structures whose elements are of relatively simple configuration. In the case of individual elements of nontrivial shape, it is still necessary to use the approach of [1] to calculate the temperature fields in such elements and this complicates the problem. In order to construct relatively simple methods for investigating the thermal regime of individual elements, this paper considers an algorithm for calculating temperature fields in thin-walled structural elements of given configuration.

Figure 1 shows individual thin-walled structural elements in which the temperature can change both along the z coordinate and within element sections for which the z coordinate is a normal because of the thermal action of the environment or other factors.

Before writing down the mathematical formulation of the problem, we give some definitions. Let D be the spatial region in which the distribution of the temperature T is sought. $D_j \in D$ is a subregion of the region D in which the temperature distribution is described by the traditional, and two-dimensional in this case, equations of thermal conductivity. In each region D_j we introduce an orthogonal coordinate system (z, x_j), $j = 1, 2, \ldots, N$. Note that the z coordinate is common to all D_j and the x_j are parallel to any section for which z is a

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 33, No. 6, pp. 1037-1041, December, 1977. Original article submitted April 4, 1977.

0022-0841/77/3306-1447$07.50

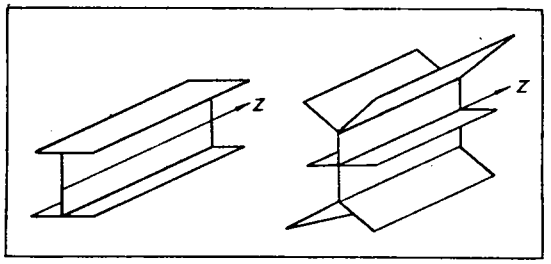

Fig. 1. Thin-walled structural elements.

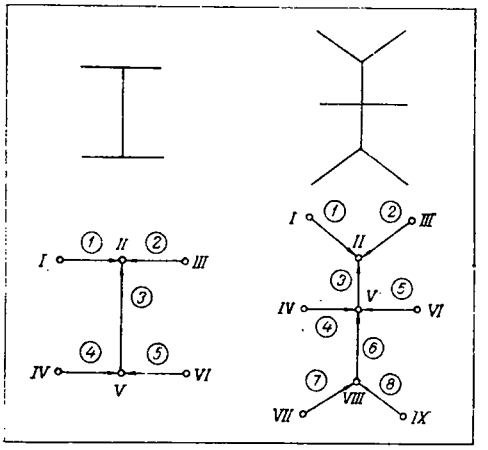

Fig. 2. Sequence for combining the regions D_j into the region D.

normal. The subregion D^c of the region D, which is obtained by sectioning the region D by means of a plane perpendicular to z, is called a contour of the region D. The contour D^c shows the sequence for combining the regions D_j into the region D (see Fig. 2). Corresponding to the contour D^c, we introduce the oriented graph G'(V) for the contour, on which the coordinate system x_j, j = 1, 2, ..., N, is given. Each j-th branch of the graph G'(V) (i.e., the corresponding subregion $D_j^c \in D^c$) and the direction x_j assigned on it completely determine the coordinate x_j for calculating the desired temperature function in the region D_j (j = 1, 2, ..., N).

We divide the set of vertices V of the graph G'(V) into two subsets: 1) the subset V_b of boundary graph vertices in which the heat-transfer boundary conditions between the structural element and the environment are given, and 2) the subset V_i of internal graph vertices for which thermal coupling conditions are given.

We formulate the boundary-value problem for the calculation of temperature distribution in a given structural element in the following way with the definitions given being taken into consideration. We seek a solution in the region D for the equation of thermal conductivity:

$$\rho(z, x_\eta, T) C_p(z, x_\eta, T) \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(\lambda(z, x_\eta, T) \frac{\partial T}{\partial z}\right) + \frac{\partial}{\partial x_j}\left(\lambda(z, x_j, T) \frac{\partial T}{\partial x_j}\right) \delta(\eta - j) + q_V(t, z, x_\eta), \tag{1}$$

$$\delta(\eta - j) = \begin{cases} 1, & \eta = j, \\ 0, & \eta \neq j, \end{cases} \tag{2}$$

$$\eta, j = 1, 2, \ldots, N. \tag{3}$$

The initial conditions are

$$T(z, x_j)|_{t=0} = T_0(z, x_j), \; j = 1, 2, \ldots, N. \tag{4}$$

Boundary conditions are

$$\lambda(z, x_j, T) \frac{\partial T}{\partial n_z}\Big|_{B_z} = q_{B_z}, \quad j = 1, 2, \ldots, N, \tag{5}$$

$$\lambda(z, x_j, T) \frac{\partial T}{\partial n_{x_j}}\Big|_{B_{x_j}} = q_{B_{x_j}}, \quad B_{x_j} \in V_B, \quad j = 1, 2, \ldots, N. \tag{6}$$

The thermal coupling conditions on $V_i \cap D$ are

$$T(z, x_j)|_\alpha = \text{const}_\alpha(t, z, x_j), \tag{7}$$

$$\sum_{k=1}^{N_\alpha} f_k \lambda_k \frac{\partial T}{\partial x_k} + q_V(t, z, x_k) = m_\alpha C_{p_\alpha} \frac{\partial T_\alpha}{\partial t}. \tag{8}$$

Here

$$\alpha \in V_n,\ \alpha = 1, 2, \ldots, N_n. \tag{9}$$

The source function $q_V(t, z, x_\eta)$ in Eq. (1) characterizes volume heat release and also heat transfer between the given element and the environment.

We use certain properties of local one-dimensional schemes [2] in constructing an algorithm for solving the problem. We seek an approximate solution $\hat{T}$ for Eq. (1) at $t = t_{k+1}$, solving the following problems written in operator form sequentially along the coordinates x_j and z.

Problem I

$$\frac{\partial T_j^I}{\partial t} = L_{x_j} T_j^I + q_V^I,\ T_j^I \in D_j, \tag{10}$$

$$T_{j,0}^I = T_{j,k}, \tag{11}$$

$$\lambda \frac{\partial T_j}{\partial n_{x_j}}\Big|_{\Gamma_{x_j}} = q_{\Gamma_{x_j}},\ B_{x_j} \in V_b, \tag{12}$$

$$T_j^I|_\alpha = \mathrm{const}_\alpha(t, z, x_j), \tag{13}$$

$$\sum_{k=1}^{N_\alpha} f_k \lambda_k \frac{\partial T_j}{\partial x_k} + q(t, z, x_k) = m_\alpha C_{p\alpha} \frac{\partial T_\alpha}{\partial t}. \tag{14}$$

$$\alpha \in V_n,\ \alpha = 1, 2, \ldots, N_n, \tag{15}$$

$$j = 1, 2, \ldots, N. \tag{16}$$

Problem II

$$\frac{\partial T^{II}}{\partial t} = L_z T^{II} + q_V^{II},\ T^{II} \in D, \tag{17}$$

$$T_0^{II} = T_j^I,\ j = 1, 2, \ldots, N, \tag{18}$$

$$\lambda \frac{\partial T}{\partial n_z}\Big|_{B_z} = q_{B_z}. \tag{19}$$

Here

$$q_V^I + q_V^{II} = q_V(t, z, x_\eta),\ \eta = 1, 2, \ldots, N. \tag{20}$$

Problem I corresponds to calculation of the temperature distribution in sections of a given element which are perpendicular to the z coordinate, i.e., to the calculation of the temperature distribution on graphs of the contour of a given element. Temperature variation along the z coordinate is determined from a solution of problem II.

We replace the equations in the system (10)-(16) and the equation for the problem (17)-(19) by their difference analogs resulting from a finite-difference approximation in the space-time mesh:

$$\bar{\omega}_{ht} = \{z = \eta h_z,\ \eta = 0, 1, 2, \ldots, N_z;\ x_j = i h_{x_j},$$
$$i = 0, 1, 2, \ldots, N_{x_j},\ j = 1, 2, \ldots, N;\ t = k h_t,$$
$$k = 1, 2, \ldots, N_t\}. \tag{21}$$

In this case we use a two-level six-point pattern with a weight σ^p (p = I, II), the value of which makes it possible to use an implicit scheme for solution.

We then find that for solution of the original problem it is first necessary to solve in each section of an element, i.e., for each $z \in \bar{\omega}_{ht}$, a system of N difference equations of the form

$$\frac{\hat{T}_j^I - T_j}{h_t} = \Lambda_{x_j} \hat{T}_j + \hat{q}_V^I, \tag{22}$$

$$j = 1, 2, \ldots, N, \tag{23}$$

assigned on the contour graph of a structural element with appropriate initial and boundary conditions. A method for solution of algebraic systems of difference equations with three-diagonal matrices given on the graph was proposed in [3] and used in [4, 5] for solution of this system. A solution of the equation system (22) is in fact a step along the primary direction. Then for each $x_j \in \bar{\omega}_{ht}$, j = 1, 2, ..., N, the difference equation

$$\frac{\hat{T}_j^{11} - \hat{T}_j^1}{h_t} = \Lambda_z \hat{T}_j^{11} + \hat{q}_V^{11}, \tag{24}$$

$$j = 1, 2, \ldots, N, \tag{25}$$

is solved. Here, the traditional stepping method is used. Through solution of Eq. (24) a step is taken along the secondary direction and finally the temperature distribution in a given element is determined.

Thus the solution of the original problem, as in local one-dimensional schemes, is reduced to a step-by-step solution of the problem of calculating the temperature fields along each coordinate (along each direction). In contrast to local one-dimensional schemes, however, a step along one of the directions in this algorithm determines not merely the temperature change along one coordinate but the temperature distribution on the graph of an element contour.

In conclusion, we briefly formulate the order of solution of the original problem.

The direction of the longitudinal z axis is selected.

The contour of an element is determined and the oriented graph of the contour constructed.

The finite-difference approximation for differential operators is realized.

Problem (22) for calculation of the temperature distribution on the contour of an element for each $z \in \bar{\omega}_{ht}$ is solved.

Problem (24) for calculation of the axial temperature distribution in an element is solved.

Note that since the use of local one-dimensional schemes assumes equivalence in the selection of direction, one can first consider the solution of problem (24) (step along the primary direction) and then solution of problem (22) (step along the secondary direction).

NOTATION

T, temperature; t, time; x_j, z, spatial coordinates; subscripts of operators corresponding to these coordinates; n, normal; LT, parabolic differential operator; ΛT, finite-difference analog of LT; q_V, source function; D, region in which solution of original heat-conduction equation is sought; B, boundary of region D, subscript for boundary of appropriate region; λ, coefficient of thermal conductivity; ρ, density; C_p, heat capacity; $\bar{\omega}_{ht}$, space-time mesh; h, mesh step; f, contact area of joined branches (for inner graph vertices); V, set of graph vertices; V_b, set of boundary graph vertices; V_i, set of inner graph vertices; N, number of branches on graph G'(V); N_i, number of inner graph vertices; N_α, number of graph branches converging at vertex α; N_z, number of mesh points in the $\bar{\omega}_{ht}$ mesh in the z direction; N_{xj}, number of mesh points in the $\bar{\omega}_{ht}$ mesh on the j-th branch of the contour graph; σ, weight of difference scheme; T_j, value of grid function at time t_k; $\hat{T}_j$, value of the grid function at time t_{k+1}; α, subscript for graph vertex; j, η, subscripts of graph fin; k, time subscript.

LITERATURE CITED

1. V. S. Khokhulin and B. M. Pankratov, "Method for calculating multidimensional temperature fields in structures," Heat and Mass Transfer V, Vol. IX. Theory of Thermal Conductivity [in Russian], ITMO, Akad. Nauk BSSR, Minsk (1976).
2. A. A. Samarskii, Introduction to the Theory of Difference Schemes [in Russian], Nauka, Moscow (1971).
3. I. V. Fryazinov, "Algorithm for graphical solution of difference problems," Zh. Vychisl. Mat. Mat. Fiz., 10, No. 2 (1970).
4. V. S. Khokhulin, "Method for studying thermal regime of structures of arbitrary configuration," Inzh.-Fiz. Zh., 29, No. 1 (1975).

5. I. S. Zhitomirskii, L. A. Borisenko, L. A. Ishchenko, and V. A. Pestryakov, "Graphical problems in nonstationary heat transfer and methods for their solution," Heat Transfer, V, Vol. VII. Heat Transfer by Radiation and Complex Heat Transfer [in Russian], ITMO, Akad. Nauk BSSR, Minsk (1976).

RECONSTRUCTION OF LOCAL EQUILIBRIUM TEMPERATURE FIELDS IN AN EMISSIVE MEDIUM

V. V. Pikalov and N. G. Preobrazhenskii

UDC 533.9

A spectral method for determination of local temperatures in an emitting volume is discussed. The problem of reconstruction of emissivity in the case of a medium of arbitrary configuration is solved by regularization.

In a number of thermophysical problems, it is often necessary to determine temperature fields within an emitting volume of plasma or of a high-temperature gas flow. The use of the methods of emission and absorption spectroscopy makes it possible to obtain the necessary pyrometric information without introducing perturbations in the test medium. The procedure for finding the temperature $T(x, y)$ after determination of the emissivity $\varepsilon(x, y)$ and absorptivity $\varkappa(x, y)$ has been developed satisfactorily [1, 2], but generally the search for these functions is a complex inverse problem. Actually, it is necessary to determine the coefficients of the radiation-transport equation from values $I(S)$ of the solution of this equation measured on the boundary of the volume. The main results in this problem were obtained with reference to the particular case of axial symmetry where the problem becomes one-dimensional. If the absorptivity is negligibly small, (optically thin layer), the problem reduces to a solution of the Abelian integral equation [1]

$$I(x) = 2\int_x^R \frac{\varepsilon(r)\,r\,dr}{\sqrt{r^2 - x^2}}, \tag{1}$$

where R is the radius of the emitting volume. However, cases with elliptical symmetry in $\varepsilon(x, y)$ can also be reduced to such an equation. Let the orientation of an ellipse with semiaxes a and b be characterized by the parameter t:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{t^2}{a^2}, \quad b < a, \tag{2}$$

$$x \in [-a, a],\ y \in [-b, b],\ t \in [0, a].$$

Making measurements along the y axis, we obtain

$$I(x) = 2\int_0^{b\sqrt{1-\frac{x^2}{a^2}}} \varepsilon(x, y)\,dy = 2\frac{b}{a}\int_x^a \frac{\varepsilon(t)\,t\,dt}{\sqrt{t^2 - x^2}}, \tag{3}$$

i.e., once again an Abelian equation but with respect to the isolines of an ellipse rather than a circle as in Eq. (1). A deficiency of such a treatment of elliptical symmetry is the need for preliminary experimental determination of the orientation of the test elliptical object in the laboratory coordinate system.

A large amount of work was devoted to solution of the Abelian equation by various methods including the use of regularization of one kind or another [3-6]. A comparison was made [7] of a number of methods with respect to the intensification of the experimental errors in them.

In the general case, the lack of symmetry in the problem is expressed in the form of an integral equation of the first kind:

Institute of Theoretical and Applied Mechanics, Siberian Branch of the Academy of Sciences of the USSR, Novosibirsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 33, No. 6, pp. 1042-1046, December, 1977. Original article submitted April 5, 1977.

0022-0841/77/3306-1451 $07.50